C34

Further probability

CHAPTER ASSIGNMENT · 13 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Give probabilities as fractions in their simplest form unless a question says otherwise. Draw tree diagrams and tables on the lined space or on spare paper.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 34 (Further probability), for classroom use. Where a question depends on a picture in the book, the picture's data is given in words or a table, and the question says so. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 34.1 · Combined events

34.1.1 (a) Two fair, tetrahedral dice are rolled. If each is numbered 1–4, draw a two-way table to show all the possible outcomes. (b) What is the probability that both dice show the same number? (c) What is the probability that the number on one dice is double the number on the other? (d) What is the probability that the sum of both numbers is prime?

34.1.2 Two fair dice are rolled. Copy and complete the diagram to show all the possible combinations (a 6 by 6 two-way table with Dice 1 across the top and Dice 2 down the side, each cell showing the pair, such as 3,5). What is the probability of getting: (a) a double 3, (b) any double, (c) a total score of 11, (d) a total score of 7, (e) an even number on both dice, (f) an even number on at least one dice, (g) a total of 6 or a double, (h) scores which differ by 3, (i) a total which is either a multiple of 2 or 5?

Exercise 34.2 · Tree diagrams

34.2.1 (a) A computer uses the numbers 1, 2 or 3 at random to make three-digit numbers. Assuming that a number can be repeated, show on a tree diagram all the possible combinations that the computer can print. (b) Calculate the probability of getting: (i) the number 131, (ii) an even number, (iii) a multiple of 11, (iv) a multiple of 3, (v) a multiple of 2 or 3.

34.2.2 (a) A cat has four kittens. Draw a tree diagram to show all the possible combinations of males and females. [assume P (male) = P (female)] (b) Calculate the probability of getting: (i) all female, (ii) two females and two males, (iii) at least one female, (iv) more females than males.

34.2.3 (a) A netball team plays three matches. In each match the team is equally likely to win, lose or draw. Draw a tree diagram to show all the possible outcomes over the three matches. (b) Calculate the probability that the team: (i) wins all three matches, (ii) wins more times than loses, (iii) loses at least one match, (iv) either draws or loses all three matches. (c) Explain why it is not very realistic to assume that the outcomes are equally likely in this case.

34.2.4 A spinner is split into quarters as shown. The four quarters are coloured pink, light blue, dark blue and white (the picture is in the book, page 490). (a) If it is spun twice, draw a probability tree showing all the possible outcomes. (b) Calculate the probability of getting: (i) two dark blues, (ii) two blues of either shade, (iii) a pink and a white in any order.

Exercise 34.3 · Tree diagrams with unequal probabilities

34.3.1 A board game involves players rolling a dice. However, before a player can start, each player needs to roll a 6. (a) Copy and complete the tree diagram showing all the possible combinations for the first two rolls of the dice. The tree has Roll 1 and Roll 2, each branch Six or Not six, with 1/6 on each Six branch and 5/6 on each Not six branch; the first outcome is Six, Six with probability 1/6 × 1/6 and the next is Six, Not six with probability 1/6 × 5/6 = 5/36. (b) Calculate the probability of the following: (i) getting a six on the first roll, (ii) starting within the first two rolls, (iii) starting on the second roll, (iv) not starting within the first three rolls, (v) starting within the first three rolls. (c) If you add the answers to b iv and v what do you notice? Explain.

34.3.2 In Italy 3/5 of the cars are foreign made. By drawing a tree diagram and writing the probabilities next to each of the branches, calculate the following probabilities: (a) the next two cars to pass a particular spot are both Italian, (b) two of the next three cars are foreign, (c) at least one of the next three cars is Italian.

34.3.3 The probability that a morning bus arrives on time is 65%. (a) Draw a tree diagram showing all the possible outcomes for three consecutive mornings. (b) Label your tree diagram and use it to calculate the probability that: (i) the bus is on time on all three mornings, (ii) the bus is late the first two mornings, (iii) the bus is on time two out of the three mornings, (iv) the bus is on time at least twice.

34.3.4 A normal pack of 52 cards is shuffled and three cards are picked at random. Draw a tree diagram to help calculate the probability of picking: (a) two clubs first, (b) three clubs, (c) no clubs, (d) at least one club.

34.3.5 A bowl of fruit contains one kiwi fruit, one banana, two mangos and two lychees. Two pieces of fruit are chosen at random and eaten. (a) Draw a probability tree showing all the possible combinations of the two pieces of fruit. (b) Use your tree diagram to calculate the probability that: (i) both the pieces of fruit eaten are mangos, (ii) a kiwi fruit and a banana are eaten, (iii) at least one lychee is eaten.

34.3.6 Light bulbs are packaged in cartons of three. 10% of the bulbs are found to be faulty. Calculate the probability of finding two faulty bulbs in a single carton.

34.3.7 A volleyball team has a 0.25 chance of losing a game. Calculate the probability of the team achieving: (a) two consecutive wins, (b) three consecutive wins, (c) 10 consecutive wins.

Exercise 34.4 · Conditional probability

34.4.1 Each of the boys and girls in a class are asked how many brothers and sisters they have. The results are shown in the table. The table headings are the number of brothers and sisters (0, 1, 2, 3, 4 or more). If a student is picked at random, calculate the probability: (a) that it is a girl, (b) that it is a boy with two or more brothers or sisters, (c) that they have no brothers or sisters given that it is a girl, (d) that it is a boy given that they have four or more brothers or sisters.

A	0	1	2	3	4 or more	Total
Boys	2	3	3	2	0	10
Girls	3	1	6	3	1	14
Total	5	4	9	5	1	24

34.4.2 The Venn diagram shows the type of vehicle owned by a group of sixty people, where C = {car}, M = {motorbike} and B = {bicycle}. In words, the diagram (a picture in the book, page 495) has these numbers of people in each region: car only 12; car and motorbike only 2; motorbike only 4; all three 3; car and bicycle only 12; motorbike and bicycle only 1; bicycle only 26; none of the three 0. If a person is picked at random, calculate the probability that they own the following vehicle type. (a) P(C) (b) P(M ∪ B) (c) P(C ∩ B) (d) P(B | C) (e) P(C | B) (f) P(C ∩ M | B) (g) P(M | C′) (h) P(M′ | B)

34.4.3 Over the course of a season, a volleyball player records how often he is selected to play for his team and whether or not his team wins. At the end of the season, he analyses the results and finds that the probability of being selected was 0.8. If he was selected, the probability of the team winning was 0.65; if he wasn't selected, the probability of the team winning was 0.45. (a) Draw a tree diagram to represent this information. (b) If the team played 100 matches during the season, calculate how many they won. (c) Given that he played in a match, calculate the probability that the team won. (d) Given that the team won a match, calculate the probability that he played.

34.4.4 The numbers 1–15 are arranged into three sets as follows: A = {odd numbers}, B = {prime numbers} and C = {multiples of two}. (a) Copy and complete the Venn diagram (three overlapping circles inside a rectangle ξ), labelling each circle and then placing each number in the correct region. (b) If a number is picked at random, calculate the following probabilities: (i) P(B) (ii) P(B ∩ C) (iii) P(A ∩ C) (iv) P(A | B) (v) P(B | C) (vi) P(C | B) (vii) P(B | B ∩ C) (c) Explain why two of the circles do not overlap each other.

Student assessment 1

SA1.1 Two normal and fair dice are rolled and their scores added together. (a) Using a two-way table, show all the possible scores that can be achieved. (b) Using your two-way table, calculate the probability of getting: (i) a score of 12, (ii) a score of 7, (iii) a score less than 4, (iv) a score of 7 or more. (c) Two dice are rolled 180 times. In theory, how many times would you expect to get a total score of 6?

SA1.2 A spinner is numbered as shown. Its eight equal sections are numbered 1, 1, 2, 2, 2, 3, 4, 5 (the picture is in the book, page 496). (a) If it is spun once, calculate the probability of getting: (i) a 1, (ii) a 2. (b) If it is spun twice, calculate the probability of getting: (i) a 2 followed by a 4, (ii) a 2 and a 4 in any order, (iii) at least one 1, (iv) at least one 2.

SA1.3 Two spinners are coloured as shown (the pictures are in the book, page 496; look at them for the colours of each section). (a) They are both spun. Draw and label a tree diagram showing all the possible outcomes. (b) Using your tree diagram, calculate the probability of getting: (i) two blues, (ii) two whites, (iii) a white and a pink, (iv) at least one white.

SA1.4 Two spinners are labelled as shown: the first has sections A, B, C and D, and the second has sections P, Q and R (the pictures are in the book, page 496). Calculate the probability of getting: (a) A and P, (b) A or B and R, (c) C but not Q.

SA1.5 A vending machine accepts $1 and $2 coins. The probability of a $2 coin being rejected is 0.2. The probability of a $1 coin being rejected is 0.1. A sandwich costing $3 is bought. Calculate the probability of getting a sandwich first time if: (a) one of each coin is used, (b) three $1 coins are used.

SA1.6 A biased coin is tossed three times. On each occasion, the probability of getting a head is 0.6. (a) Draw a tree diagram to show all the possible outcomes after three tosses. Label each branch clearly with the probability of each outcome. (b) Using your tree diagram, calculate the probability of getting: (i) three heads, (ii) three tails, (iii) at least two heads.

SA1.7 A ball enters a chute at X. The chutes are drawn in the book (page 497). In words: from X the ball goes down a left-hand chute marked 0.2 or the right-hand chute (i). The left-hand chute splits again: one way, marked 0.3, leads to tray A, and the other way, chute (ii), leads to tray B. Chute (i) also splits: one way, marked 0.7, leads to tray C, and the other way, chute (iii), leads to tray B. (a) What are the probabilities of the ball going down each of the chutes labelled (i), (ii) and (iii)? (b) Calculate the probability of the ball landing in: (i) tray A, (ii) tray C, (iii) tray B. (c) Given that the ball goes down chute (i), calculate the probability of it landing in tray B. (d) Given that the ball lands in tray B, calculate the probability that it came down chute (iii).

SA1.8 A fish breeder keeps three types of fish: A, B and C. Each type can be categorised into two sizes, small (S) and large (L). The probabilities of each are given in the tree diagram, which is described here: fish type A, B, C have probabilities 0.6, 0.3, 0.1. After A: small 0.6, large 0.4. After B: small 0.8, large 0.2. After C: small 0.1, large 0.9. (a) A fish is chosen at random. Calculate the probability that: (i) it is a large fish of type B, (ii) it is a large fish, (iii) it is a large fish given that it is of type B. (b) If a fish chosen at random is small, calculate the probability of it being of type: (i) A, (ii) B, (iii) C.

SA1.9 A football team decide to analyse their match results. They look at their probability of winning depending on whether they scored first in the game or not. The results are presented in the table. (a) Calculate the probability that the team score first. (b) Calculate the probability that the team draw. (c) Given that the team score first, calculate the probability that they draw. (d) Given that the team lose, calculate the probability that they didn't score first.

A	Win	Lose	Draw
Score first	0.38	0.12	0.18
Don't score first	0.02	0.22	0.08

Check yourself

1 A bag has 3 red and 2 blue counters. Two are taken without replacement. Draw the tree diagram and find P(both counters are the same colour). [4]

2 A coin is tossed four times. Find P(at least one head). [2]

3 Explain why the probabilities on the second pair of branches change when the first counter is not put back. [2]

4 A class has 12 boys and 18 girls. 5 of the boys and 6 of the girls walk to school. A student is chosen at random. Find P(walks | girl). [2]

5 The probability of rain is 0.3 on Monday and 0.4 on Tuesday. Find P(rain on exactly one of the two days). [3]

Chapter quiz

Q1 Two fair dice are rolled. What is the probability that the total is 7?

A	B	C	D
1/12	1/6	7/36	1/7

Q2 A fair coin is tossed three times. What is P(exactly one head)?

A	B	C	D
1/8	1/3	3/8	1/2

Q3 A fair coin is tossed three times. What is P(at least one head)?

A	B	C	D
1/8	7/8	3/8	1/2

Q4 On a tree diagram, how do you find the probability of one complete outcome?

A	B	C	D
Multiply along its branches	Add along its branches	Divide the last branch by the first	Add the branches at the end

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Q5 The probability of rain each day is 0.2. What is P(rain on both of two days)?

A	B	C	D
0.4	0.04	0.2	0.64

Q6 A bag has 3 red and 2 blue counters. Two are taken with replacement. P(both red)?

A	B	C	D
6/20	6/25	9/25	3/10

Q7 A bag has 3 red and 2 blue counters. Two are taken without replacement. P(both red)?

A	B	C	D
9/25	3/10	2/5	6/25

Q8 The outcomes at the end of a complete tree diagram are all added together. The total is:

A	B	C	D
0	1	The number of branches	It depends on the tree

Q9 What does P(A | B) mean?

A	B	C	D
A or B happens	A and B both happen	The probability of A given that B has happened	The probability of B given that A has happened

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Q10 46 girls are in a school; 34 of them study Maths. A girl is chosen. P(Maths | girl)?

A	B	C	D
17/23	17/30	34/86	30/43

Q11 Each bulb is faulty with probability 0.1. What is P(at least one faulty in two bulbs)?

A	B	C	D
0.2	0.01	0.81	0.19

Q12 A bag has 4 red and 1 blue counter. Two are taken without replacement. P(blue second)?

A	B	C	D
1/5	1/4	4/5	1/16

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-34-further-probability/