C31

Vectors

CHAPTER ASSIGNMENT

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. Here (x, y) means a column vector with x on top and y underneath. AB means the vector from A to B. Where a question gives coordinates, one square on the grid is 1 unit.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 31 (Vectors), for classroom use. The book's grid and geometry diagrams cannot be carried in this format, so those questions give the same information as coordinates, column vectors or a short description (each says so). The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 31.1 · Translations

31.1.1 Describe each translation using a column vector. The book shows A, B, C, D and E on a square grid with arrows A to B, B to C, C to D, D to E and E to A. Here are the points as coordinates: A (0, 0), B (2, 4), C (5, 5), D (3, 1), E (6, −1).

(a) AB	(b) BC	(c) CD	(d) DE	(e) EA
(f) AE	(g) DA	(h) CA	(i) DB	

31.1.2 Describe each translation using a column vector. The book shows the vectors a to e as arrows on a square grid. Here is where each arrow starts and ends: a from (1, 4) to (5, 8); b from (4, 1) to (4, 5); c from (9, 8) to (5, 8); d from (11, 1) to (9, 7); e from (9, 4) to (5, 2).

(a) a	(b) b	(c) c	(d) d	(e) e
(f) −b	(g) −c	(h) −d	(i) −a	

31.1.3 Draw and label the following vectors on a square grid: (a) a = (2, 4) · (b) b = (−3, 6) · (c) c = (3, −5) · (d) d = (−4, −3) · (e) e = (0, −6) · (f) f = (−5, 0) · (g) g = −c · (h) h = −b · (i) i = −f

Exercise 31.2 · Addition and subtraction of vectors

31.2.1 In the following questions, a = (3, 4), b = (−2, 1), c = (−4, −3), d = (3, −2). Draw vector diagrams to represent the following: (a) a + b · (b) b + a · (c) a + d · (d) d + a · (e) b + c · (f) c + b

31.2.2 What conclusions can you draw from your answers to Question 1 above?

31.2.3 Draw vector diagrams to represent the following (same a, b, c, d): (a) b − c · (b) d − a · (c) −a − c · (d) a + c − b · (e) d − c − b · (f) −c + b + d

31.2.4 Represent each of the vectors in Question 3 by a single column vector.

(a)	(b)	(c)	(d)	(e)	(f)

Exercise 31.3 · Multiplying a vector by a scalar

31.3.1 a = (1, 4), b = (−4, −2), c = (−4, 6). Express the following vectors in terms of either a, b or c. The book shows d to k as arrows on a grid. Here is each one as a column vector: d = (4, −6), e = (−1, −4), f = (2, 8), g = (−2, 3), h = (−2, −1), i = (2, 1), j = (−6, −3), k = (−1.5, −6).

(d)	(e)	(f)	(g)
(h)	(i)	(j)	(k)

31.3.2 a = (2, 3), b = (−4, −1), c = (−2, 4). Represent each of the following as a single column vector:

(a) 2a	(b) 3b	(c) −c	(d) a + b	(e) b − c
(f) 3c − a	(g) 2b − a	(h) ½(a − b)	(i) 2a − 3c	

31.3.3 a = (−2, 3), b = (0, −3), c = (4, −1). Express each of the following vectors in terms of a, b and c:

(a) (−4, 6)	(b) (0, 3)	(c) (4, −4)
(d) (−2, 6)	(e) (8, −2)	(f) (10, −5)

Exercise 31.4 · The magnitude of a vector

31.4.1 Calculate the magnitude of the vectors shown below. Give your answers correct to 1 d.p. The book shows a to f as arrows on a grid. Here is each one as a column vector: a = (5, 0), b = (4, 1), c = (−2, 4), d = (0, 7), e = (2, 7), f = (−5, 4).

(a)	(b)	(c)	(d)	(e)	(f)

(a)	(b)	(c)	(d)	(e)	(f)

31.4.2 Calculate the magnitude of the following vectors, giving your answers to 1 d.p. (a) AB = (0, 4) · (b) BC = (2, 5) · (c) CD = (−4, −6) · (d) DE = (−5, 12) · (e) 2AB · (f) 2CD

31.4.3 a = (4, −3), b = (−5, 7), c = (−1, −8). Calculate the magnitude of the following, giving your answers to 1 d.p. (a) a + b · (b) 2a − b · (c) b − c · (d) 2c + 3b · (e) 2b − 3a · (f) a + 2b − c

Exercise 31.5 · Position vectors

31.5.1 Give the position vectors of A, B, C, D, E, F, G and H relative to O in the diagram. The book shows these points on a grid with origin O. Here they are as coordinates: A (3, 3), B (1, 2), C (−3, 2), D (−3, 0), E (4, −2), F (2, −4), G (−1, −1), H (0, −3).

A	B	C	D	E	F	G	H

Exercise 31.6 · Vector geometry

31.6.1 If AG = a and AE = b, express the following in terms of a and b. The book's diagram is a rectangle ABCD with A at the top left, B top right, C bottom right and D bottom left. G, H, E and F are the midpoints of AB, DC, AD and BC, and I is the centre where GH and EF cross.

(a) EI	(b) HC	(c) FC	(d) DE	(e) GH
(f) CD	(g) AI	(h) GE	(i) FD	

31.6.2 If LP = a and LR = b, express the following in terms of a and b. The book's diagram is a triangle LMN with L at the bottom left, N at the bottom right and M at the top. P, Q and R are the midpoints of LM, MN and LN.

(a) LM	(b) PQ	(c) PR
(d) MQ	(e) MP	(f) NP

31.6.3 ABCDEF is a regular hexagon with centre G. The vertices go round in the order A (top left), B (top right), C, D, E, F. If GA = a and GB = b, express the following in terms of a and b:

(a) AD	(b) FE	(c) DC
(d) AB	(e) FC	(f) EC
(g) BE	(h) FD	(i) AE

31.6.4 If AB = a and AG = b, express the following in terms of a and b. The book's diagram has A, B, C, D, E, F equally spaced along a horizontal line (AB = a, so AF = 5a). G and K are on the line down from A with AG = GK = b, and M is below K with KM = ½b. A line joins F to M. Measuring from A in steps of a across and b down, the other points are H (1, 1), I (2, 1), J (3, 1), L (1, 2), N (2, 1½) and O (4, ½); J, L, N and O lie on the line FM.

(a) AF	(b) AM	(c) FM

(a) AF	(b) AM	(c) FM
(d) FO	(e) EI	(f) KF
(g) CN	(h) AN	(i) DN

Exercise 31.7 · Vector geometry problems

31.7.1 T is the midpoint of the line PS and R divides the line QS in the ratio 1 : 3. PT = a and PQ = b. (The book's triangle PQS has a right angle at Q, with R and S on the base QS.) (a) Express each of the following in terms of a and b: (i) PS (ii) QS (iii) PR. (b) Show that RT = ¼(2a − 3b).

31.7.2 PM = 3 LP and QN = 3LQ. (P is on the line LM and Q is on the line LN of the triangle LMN.) Prove that: (a) the line PQ is parallel to the line MN, (b) the line MN is four times the length of the line PQ.

31.7.3 PQRS is a parallelogram (P top left, Q top right). The point T divides the line PQ in the ratio 1 : 3, and U, V and W are the midpoints of SR, PS and QR respectively. X is the point where TU crosses VW. PT = a and PV = b. (a) Express each of the following in terms of a and b: (i) PQ (ii) SU (iii) PU (iv) VX. (b) Show that XR = ½(5a + 2b).

31.7.4 ABC is an isosceles triangle. L is the midpoint of BC. M divides the line LA in the ratio 1 : 5, and N divides AC in the ratio 2 : 5. (a) BC = p and BA = q. Express the following in terms of p and q: (i) LA (ii) AN. (b) Show that MN = 1/84 (46q − 11p).

Student assessment 1

SA1.1 Using the diagram, describe the following translations using column vectors. The book shows the quadrilateral ABCD with arrows A to B, B to C, C to D and D to A. Here are the points as coordinates: A (7, 3), B (5, 6), C (1, 5), D (0, 1).

(a) AB	(b) DA	(c) CA

SA1.2 Describe each of the translations shown using column vectors. The book shows a to e as arrows on a grid. Here is where each arrow starts and ends: a from (1, 3) to (3, 7); b from (4, 7) to (8, 7); c from (8, 7) to (8, 1); d from (7, 5) to (3, 3); e from (5, 1) to (3, 2).

(a) a	(b) b	(c) c	(d) d	(e) e

SA1.3 Using the vectors in Question 2, draw diagrams to represent: (a) a + b · (b) e − d · (c) c − e · (d) 2e + b

SA1.4 In the following, a = (2, 6), b = (−3, −1), c = (−2, 4). Calculate:

(a) a + b	(b) c − b	(c) 2a + b	(d) 3c − 2b

Student assessment 2

SA2.1 (a) Calculate the magnitude of the vector AB shown in the diagram. The book shows AB as an arrow on a grid. B is at (0, 0) and A is at (6, 4). (b) Calculate the magnitude of the following vectors: a = (2, 9), b = (−7, −4), c = (−5, 12).

SA2.2 p = (3, 2), q = (−4, 1), r = (3, −4). Calculate the magnitude of the following, giving your answers to 3 s.f. (a) 3p − 2q · (b) ½r + q

SA2.3 Give the position vectors of A, B, C, D and E relative to O for the diagram. The book shows these points on a grid with origin O. Here they are as coordinates: A (2, 4), B (−1, 2), C (−3, −1), D (0, −3), E (1, −4).

A	B	C	D	E

SA2.4 The book's diagram is a triangle ABC with B at the top. D is the midpoint of AB, E is the midpoint of BC and F is the midpoint of AC. The lines DE, DF and EF are drawn. (a) Name another vector equal to DE in the diagram. (b) Express DF in terms of BC. (c) Express CF in terms of DE.

Student assessment 3

SA3.1 In the triangle PQR, the point S divides the line PQ in the ratio 1 : 3, and T divides the line RQ in the ratio 3 : 2. PR = a and PQ = b. (The book's triangle has P at the top, R below P and Q to the right of R.) (a) Express the following in terms of a and b: (i) PS (ii) SR (iii) TQ. (b) Show that ST = 1/20 (8a + 7b).

SA3.2 In the triangle ABC, the point D divides the line AB in the ratio 1 : 3, and E divides the line AC also in the ratio 1 : 3. (The book's triangle has B above A and C to the right of A.) If AD = a and AE = b, prove that: (a) BC = 4 DE, (b) BCED is a trapezium.

SA3.3 The parallelogram ABCD shows the points P and Q dividing each of the lines AD and DC in the ratio 1 : 4 respectively. R is the point where PQ crosses DB. (a) If DA = a and DC = b, express the following in terms of a and b: (i) AC (ii) CB (iii) DB. (b) (i) Find the ratio in which R divides DB. (ii) Find the ratio in which R divides PQ.

Check yourself

C1 The vector from A to B is (3, −2). Write down BA, and describe the move AB in words.

C2 a = (4, 1) and b = (−2, 3). Work out (a) a + b (b) a − b (c) 2a − 3b.

C3 Calculate the magnitude of (6, −8), and the magnitude of (5, 2) to 1 d.p.

C4 OACB is a parallelogram with OA = a and OB = b. M is the midpoint of AC. Write in terms of a and b: (a) OC (b) OM (c) BM.

C5 Show that the vectors (6, −9) and (−4, 6) are parallel, and say which is longer.

Chapter quiz

Q1 The column vector (2, −3) describes a translation of what?

A	B	C	D
2 left, 3 down	2 right, 3 up	2 right, 3 down	3 right, 2 down

Q2 AB = (4, −1). What is BA?

A	B	C	D
(4, −1)	(−4, 1)	(−4, −1)	(1, −4)

Q3 a = (2, 5) and b = (−3, −2). What is a + b?

A	B	C	D
(5, 7)	(−1, 3)	(−1, 7)	(5, 3)

Q4 $\mathbf{a} = (2, 5)$ and $\mathbf{b} = (-3, -2)$. What is $\mathbf{a} - \mathbf{b}$?

A	B	C	D
$(-1, 3)$	$(-5, -7)$	$(5, 7)$	$(5, 3)$

Q5 $\mathbf{a} = (-2, 4)$. What is $3\mathbf{a}$?

A	B	C	D
$(1, 7)$	$(-6, 12)$	$(-6, 4)$	$(6, -12)$

Q6 Which vector is parallel to $(2, -4)$?

A	B	C	D
$(-1, 2)$	$(4, 2)$	$(2, 4)$	$(1, 2)$

Q7 What is the magnitude of the vector $(3, 4)$?

A	B	C	D
5	7	25	12

Q8 What is the magnitude of $(-5, 12)$?

A	B	C	D
7	13	17	169

Q9 $\overrightarrow{AB} = (3, 1)$ and $\overrightarrow{BC} = (-1, 4)$. What is $\overrightarrow{AC}$?

A	B	C	D
$(4, -3)$	$(2, 5)$	$(-2, 5)$	$(4, 5)$

Q10 A point P is at $(-3, 2)$ on a grid with origin O. What is the position vector $\overrightarrow{OP}$?

A	B	C	D
$(3, -2)$	$(2, -3)$	$(-3, 2)$	$(-2, 3)$

Q11 M is the midpoint of AB and AM = a. Which is AB?

A	B	C	D
½a	2a	a + 2	−a

Q12 The vector a has magnitude 5. What is the magnitude of −2a?

A	B	C	D
−10	3	7	10

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-31-vectors/