C15

Sequences

CHAPTER ASSIGNMENT · 227 MARKS

Name _____ Class _____ No. _____ Date _____

Answer every question in the space given. u_n is the nth term and u_{n+1} is the term after it. Show your table of differences where you use one. You may use a calculator for Exercise 15.4 and Student assessment 2.

Practice questions from *Cambridge IGCSE Mathematics Core and Extended* (4th edition), Chapter 15 (Sequences), for classroom use. Every book exercise and Student assessment question is included. The Check yourself and Chapter quiz questions are Guru Jazzy's own.

Exercise 15.1 · Arithmetic sequences

15.1.1 For each of the following sequences, the term-to-term rule and one of the terms are given. Calculate the terms required and state whether the sequence generated is arithmetic or not. [12]

(a) $u_{n+1} = u_n + 5$, where $u_1 = 2$. Calculate u_2, u_3 and u_4

(b) $u_{n+1} = u_n - 3$, where $u_1 = 4$. Calculate u_2, u_3 and u_4

(c) $u_{n+1} = u_n - 4$, where $u_3 = 2$. Calculate u_1, u_2 and u_4

(d) $u_{n+1} = 2u_n - 1$, where $u_2 = 0$. Calculate u_1, u_3 and u_4

(e) $u_{n+1} = u_n/2 + 3$, where $u_1 = 2$. Calculate u_2, u_3 and u_4

(f) $u_{n+1} = u_n/2 - 1$, where $u_3 = 10$. Calculate u_1, u_2 and u_4

15.1.2 For each of the following sequences: (i) describe the term-to-term rule in words (ii) write the term-to-term rule in its general form using the notation u_{n+1} and u_n (iii) deduce the formula for the nth term (iv) calculate the 10th term. [24]

(a) 5, 8, 11, 14, 17

(b) 0, 4, 8, 12, 16

(c) ½, 1½, 2½, 3½, 4½

(d) 6, 3, 0, −3, −6

(e) −7, −4, −1, 2, 5

(f) $-9, -13, -17, -21, -25$

15.1.3a Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1	2	5		50	n
Term				45		4n − 3

15.1.3b Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1	2	5			n
Term				59	449	6n − 1

15.1.3c Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1				100	n
Term		0	−5	−47		−n + 3

15.1.3d Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1	2	3			n
Term	3	0	−3	−24	−294	

15.1.3e Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position		5	7			n
Term	1	10	16	25	145	

15.1.3f Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1	2	5		50	n
Term	−5.5	−7		−34		

15.1.4 For each of the following arithmetic sequences: (i) deduce the common difference d (ii) write the term-to-term rule in its general form using the notation u_{n+1} and u_n (iii) give the formula for the nth term (iv) calculate the 50th term. [24]

(a) 5, 9, 13, 17, 21

(b) 0, ..., 2, ..., 4

(c) −10, ..., ..., ..., 2

(d) $u_1 = 6$, $u_9 = 10$

(e) $u_3 = -50$, $u_{20} = 18$

(f) $u_5 = 60$, $u_{12} = 39$

Exercise 15.2 · Quadratic rules

15.2 By using a table if necessary, find the formula for the nth term of each of the following sequences. [20]

(1) 2, 5, 10, 17, 26

(2) 0, 3, 8, 15, 24

(3) 6, 9, 14, 21, 30

(4) 9, 12, 17, 24, 33

(5) −2, 1, 6, 13, 22

(6) 4, 10, 20, 34, 52

(7) 0, 6, 16, 30, 48

(8) 5, 14, 29, 50, 77

(9) 0, 12, 32, 60, 96

(10) 1, 16, 41, 76, 121

Exercise 15.3 · Cubic rules

15.3 Use a table to find the formula for the nth term of the following sequences. [24]

(1) 11, 18, 37, 74, 135

(2) 0, 6, 24, 60, 120

(3) −4, 3, 22, 59, 120

(4) 2, 12, 36, 80, 150

(5) 7, 22, 51, 100, 175

(6) 7, 28, 67, 130, 223

(7) 1, 10, 33, 76, 145

(8) 13, 25, 49, 91, 157

Exercise 15.4 · Exponential sequences (Extended)

15.4.1 Identify which of the following are exponential sequences and which are not. [6]

(a) 2, 6, 18, 54

(b) 25, 5, 1, 1/5

(c) 1, 4, 9, 16

(d) −3, 9, −27, 81

(e) 1/2, 2/3, 3/4, 4/5

(f) 1/2, 2/4, 4/8, 8/16

15.4.2 For the sequences in question 1 that are exponential, calculate: (i) the common ratio r (ii) the next two terms (iii) a formula for the nth term. [9]

15.4.3 The nth term of an exponential sequence is given by the formula $u_n = -6 \times 2^{n-1}$.

(a) Calculate u_1, u_2 and u_3. [3]

(b) What is the value of n if $u_n = -768$? [2]

15.4.4 Part of an exponential sequence is given below: ..., −1, ..., ..., 64, ... where $u_2 = -1$ and $u_5 = 64$. Calculate:

(a) the common ratio r [2]

(b) the value of u_1 [1]

(c) the value of u_{10}. [2]

15.4.5 A homebuyer takes out a loan with a mortgage company for $200 000. The interest rate is 6% per year. If she is unable to repay any of the loan during the first 3 years, calculate the extra amount she will have to pay because of interest by the end of the third year. [3]

15.4.6 A car is bought for $10 000. It loses value at a rate of 20% each year.

(a) Explain why the car is not worthless after 5 years. [1]

(b) Calculate its value after 5 years. [2]

(c) Explain why a depreciation of 20% per year means, in theory, that the car will never be worthless. [2]

Exercise 15.5 · Combinations of sequences (Extended)

15.5 In each of the questions below: (i) write down the rule for the nth term of the sequence by inspection (ii) write down the next two terms of the sequence. [24]

(1) 2, 5, 10, 17

(2) 3, 10, 29, 66

(3) ½, 2, 4½, 8

(4) 1, 2, 4, 8

(5) 2, 6, 12, 20

(6) 2, 12, 36, 80

(7) 0, 4, 18, 48

(8) 6, 12, 24, 48

Student assessment 1

SA1.1 For each of the sequences given below: (i) calculate the next two terms, (ii) explain the pattern in words. [12]

(a) 9, 18, 27, 36, ...

(b) 54, 48, 42, 36, ...

(c) 18, 9, 4.5, ...

(d) 12, 6, 0, −6, ...

(e) 216, 125, 64, ...

(f) 1, 3, 9, 27, ...

SA1.2 For each of the sequences shown below give an expression for the nth term: [12]

(a) 6, 10, 14, 18, 22, ...

(b) 13, 19, 25, 31, ...

(c) 3, 9, 15, 21, 27, ...

(d) 4, 7, 12, 19, 28, ...

(e) 0, 10, 20, 30, 40, ...

(f) 0, 7, 26, 63, 124, ...

SA1.3 For each of the following arithmetic sequences: (i) write down a formula for the nth term (ii) calculate the 10th term. [4]

(a) 1, 5, 9, 13, ...

(b) 1, −2, −5, −8, ...

SA1.4 For both of the following, calculate u_5 and u_{100}: [4]

(a) $u_n = 6n - 3$

(b) $u_n = -\frac{1}{2}n + 4$

SA1.5a Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e	f
Position	1	2	3	10		n
Term	17	14			−55	

SA1.5b Copy and complete the table of an arithmetic sequence.

Column	a	b	c	d	e
Position	2	6	10		n
Term	−4	−2		35	

Student assessment 2 (Extended)

SA2.1 A girl deposits $300 in a bank account. The bank offers 7% interest per year. Assuming the girl does not take any money out of the account, calculate:

(a) the amount of money in the account after 8 years [2]

(b) the minimum number of years the money must be left in the account, for the amount to be greater than $350. [2]

SA2.2 A computer loses 35% of its value each year. If the computer cost $600 new, calculate:

(a) its value after 2 years [2]

(b) its value after 10 years. [2]

SA2.3 Part of an exponential sequence is given below: ..., ..., 27, ..., ..., −1 where $u_3 = 27$ and $u_6 = -1$. Calculate:

(a) the common ratio r [2]

(b) the value u_1 [1]

(c) the value of n if $u_n = -1/81$ [2]

SA2.4 Using a table of differences if necessary, calculate the rule for the nth term of the sequence 8, 24, 58, 116, 204 ... [4]

SA2.5 Using a table of differences, calculate the rule for the nth term of the sequence 10, 23, 50, 97, 170 ... [4]

Check yourself

1 Explain the difference between a term-to-term rule and a formula for the nth term. Use the sequence 4, 9, 14, 19, ... in your answer. [3]

2 Is 100 a term of the sequence $u_n = 3n + 2$? Show how you decide. [2]

3 The 2nd differences of a sequence are all 6. What is the coefficient of n^2 in its nth term? Explain why. [2]

4 Find the nth term of 3, 6, 11, 18, 27, ... and use it to find the 20th term. [3]

5 An exponential sequence has $u_1 = 80$ and $r = ½$. Write its first four terms and explain why no term is ever 0. [3]

Chapter quiz

Q1 $u_{n+1} = u_n + 6$ and $u_1 = 4$. What is u_3?

A	B	C	D
10	16	22	24

Q2 What is the common difference of 11, 8, 5, 2, ...?

A	B	C	D
3	−3	−2	8

Q3 Find the nth term of 5, 9, 13, 17, ...

A	B	C	D
4n + 1	4n + 5	n + 4	5n + 4

Q4 $u_n = 3n - 7$. What is the 20th term?

A	B	C	D
53	60	67	−4

Q5 Is 75 a term of $u_n = 4n + 2$?

A	B	C	D
Yes, n = 18	Yes, n = 19	No, n would not be whole	No, the terms are even

Q6 The 2nd differences of a sequence are all 8. What is the n^2 coefficient?

A	B	C	D
2	4	8	16

Q7 Which row of differences is constant for a cubic sequence?

A	B	C	D
1st	2nd	3rd	4th

Q8 Find the nth term of 2, 8, 18, 32, 50, ...

A	B	C	D
$2n^2$	$n^2 + 1$	$6n - 4$	2^n

Q9 What is the common ratio of 81, 27, 9, 3, ...?

A	B	C	D
3	−3	1/3	−54

Q10 An exponential sequence has $u_1 = 5$ and $r = 2$. What is u_4?

A	B	C	D
20	40	80	11

Q11 $400 earns 5% compound interest a year. What is it worth after 2 years?

A	B	C	D
$440.00	$441.00	$420.00	$800.00

Q12 Find the nth term of 3, 5, 9, 17, 33, ...

A	B	C	D
$2^n - 1$	$2n + 1$	$n^2 + 2$	$2^n + 1$

End of assignment · Check your answers against the chapter at gurujazzy.com/subjects/o-level-mathematics/chapter-15-sequences/